Warming up for PBL: a course in mathematical modelling and problem solving for engineering students

Dag Wedelin[*, a] and Tom Adawi[b]

[a]*Department of Computer Science and Engineering, Chalmers University of Technology;* [b]*Division for Engineering Education Research, Chalmers University of Technology*

The step from traditional teaching to PBL is considerable and it has previously been proposed that students should be skilled at problem solving before entering a PBL course. In this paper, we first discuss some key ideas behind the design of a successful course in mathematical modelling and problem solving for engineering students. A central aim of the course is to help the students to understand the power of learning by exploration, a missing key component in the students' ability to solve problems. We then discuss how this kind of course can serve as an intermediate step in a progression towards more self-directed project-based and problem-based learning.

Key words: PBL, problem solving, mathematical modelling, engineering education, progression

INTRODUCTION

Engineering education has been criticized for neglecting to demonstrate the relevance of the topic being taught (Prince & Felder, 2006; Prince & Hoyt, 2002), and for neglecting to provide students with opportunities to develop skills that are essential for dealing with complex real-world problems related to science and technology (Litzinger et al., 2011; Woods et al., 1997). These issues seem to be particularly grave in the mathematical education of engineers, where students – at least in some specializations – have significant difficulties in connecting and applying what they have learnt in introductory mathematics courses (Adawi et al., 2005; Wedelin et al., 2014), sometimes leading to the belief that mathematics is not relevant to them as engineers (Flegg et al., 2012). These educational issues formed the backdrop to the development of a course in mathematical modelling and problem solving which is now being offered to second year engineering students at Chalmers University of Technology (Wedelin & Adawi, 2014).

Two renowned instructional approaches for dealing with these issues are *problem-based learning* and *project-based learning* (Mills & Treagust, 2003). A central feature of these instructional approaches is that an authentic problem "makes the starting point for the learning processes, places learning in context, and bases learning on the learner's experience" (Kolmos, Graaff & Du, 2009; p. 11). An important difference is that project-based learning is more oriented towards the application of knowledge, whereas problem-based learning is more oriented towards the acquisition of knowledge (Mills & Treagust, 2003). Moreover, in project-based learning the goal is to produce a product or artefact (van Barneveld & Strobel, 2009). At the same time, these two approaches share and embody a number of principles that support learning – apart from situating learning in an authentic problem or activity – such as *active-* and *collaborative* learning, *self-directed* learning and *interdisciplinary* learning (Graaff & Kolmos, 2003), and PBL

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has become an acronym for both problem-based and project-based learning (Kolmos, Graaff & Du, 2009). It is worth noting that the form of PBL that is practiced in engineering education is often project-based, partly due to the hierarchical nature of engineering knowledge (Gavin, 2011).

However, the step from traditional teaching to learning to handle the complex real-world problems typically addressed in PBL is considerable. It may therefore be useful to establish a *progression*, where problem-solving skills are first explicitly addressed. Prince and Hoyt (2002), for example, noted that "the traditional undergraduate engineering curriculum is not designed to systematically develop relevant problem solving skills". They argue that "students should be introduced to relevant problem solving early in the curriculum and *gradually* encouraged and trained to adopt appropriate problem solving skills". Woods et al. (1997) found that having engineering students solve interesting and open-ended problems in small groups did little to develop their problem solving skills and confidence. On the other hand, they found that workshops focusing on problem solving skills enhanced the students' skills and confidence. Woods (2014) argues that students should be skilled at problem solving before entering a PBL program, since PBL offers an opportunity to develop these skills but students do not develop problem-solving skills without "explicit interventions on the part of the teacher".

The course in mathematical modelling and problem solving aims to improve real-world problem solving in science and technology, with a particular focus on the ability to use mathematics in such problem solving. The course specifically targets two important skills that are commonly overlooked in traditional teaching, because of its focus on explaining existing models and methods. Recognizing the *ill-structured* character and diversity of real-world problems and situations, the course therefore addresses *mathematical modelling*, which we see as an ability to see relevant (mathematical) aspects and create a mathematical description, acting as an important bridge from ill-structured real-world problems to conceptual representations and theory. Likewise, *problem solving*, in terms of an ability to explore and handle situations where no known or complete method is available, greatly extends the range of future problems that can be handled. Because of its mathematical character, the course is less tied to a specific subject than most courses in science and engineering. At the same time, the focus on mathematical modelling makes it natural to reach out to different areas of application, to a greater extent than most courses in mathematics.

The course is based on a set of reasonably realistic problems related to the application of mathematics in different fields, which are solved in a controlled setting intended to be especially suited for the learning and explicit teaching of problem solving. Together, the specific course objectives and the learning environment therefore provide a lower threshold for students who have little experience in handling more complex problems. In the following, we first describe some theoretical concepts and frameworks informing the design of the course. We then more closely discuss the motivation and philosophy of the course, the design of the problems and the learning environment. Finally, we discuss how this kind of course can serve as an intermediate step in a progression towards more self-directed PBL.

THEORETICAL CONCEPTS AND FRAMEWORKS

A key idea informing the design of the course is the notion of *variation*. A fundamental observation is that problems and situations occur in never-ending minor and major variations, and it is not possible to handle this diversity by learning a specific method for each problem. Rather, we may attempt to learn to handle future variation in new problems, by practicing to handle

variation with problems that we have today (Bowden & Marton, 1998). Here, we want to emphasize the distinction between *well-structured* and *ill-structured* problems (Jonassen, 2011). Well-structured problems are well-defined and self-contained, while ill-structured problems typically are formulated in an incomplete way, where the question is not precise, and where judgement and external contextual information are important for handling the problem. Well-structured problems are common in school situations and in abstract mathematical problem solving, while real-world problems are typically more ill-structured (Jonassen, 2011).

We also draw on a framework for *mathematical thinking* developed by Schoenfeld (1985; 1992). He argues that four aspects of mathematical thinking are necessary and sufficient for understanding why individuals are successful or not when engaging in mathematical problem solving: 1) resources – the individual's mathematical knowledge base; 2) heuristics – the individual's problem solving strategies; 3) self-regulation – the individual's monitoring and controlling of thinking processes; and 4) beliefs – the individual's "understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behaviour".

Another theoretical framework that is useful for our purposes is *case-based reasoning* (CBR), describing reasoning activities and learning in problem solving situations (Kolodner, 1993; Schank, 1982). CBR is based on the assumption that there is a "preference to reason using the most *specific* and most *cohesive* applicable knowledge available [...] and cases have both of these properties" (Kolodner et al., 1996). According to CBR, expert problem solvers "assess the current problem, find previous cases relevant to the current problem, leverage that case to inform a solution, assess the potential solution, and update internal memory as one learns from the experience" (Tawfik & Keene, 2013). Access to old cases or experiences is thus at the heart of the CBR cycle. Jonassen (2011) illustrates how practitioners across a range of disciplines, including engineering, to a large degree deal with ill-structured problems by reusing old cases relevant to the problem. Kolodner et al. (1996) argue that old cases "record what is possible, providing a reasoner with more probability of moving forward in a workable way than is provided by using general knowledge that is merely plausible". (We note that the related term *case-based learning* does not seem to have a clear definition, and our focus here is primarily on case-based reasoning as a way to think when solving a problem.)

Finally, we draw on *cognitive apprenticeship* (Collins, Brown & Newman, 1989) to discuss the overall design of the learning environment. In traditional apprenticeship, the physical processes to be mastered are external and hence directly available for the students to observe. In order to apply apprenticeship methods to teach cognitive processes, these internal processes have to be made external, visible to the students, which is called modelling (not to be confused with the mathematical modelling as one of the course topics). One strategy to model the cognitive processes involved in problem solving is "think aloud modelling", where teachers "describe what they are thinking and doing, why they are doing what they are doing, and verbalize their self-correction processes" (Duncan, 1996). The content that is taught and learned using a cognitive apprenticeship approach thus goes beyond domain knowledge (declarative knowledge such as facts, concepts and procedures of a subject) to include different types of strategic knowledge or "tacit" knowledge that experts draw on when solving problems. Collins, Brown and Newman (1989) describe three types of strategic knowledge: heuristic strategies or "tricks of the trade", metacognitive strategies, and learning strategies. In cognitive apprenticeship, modelling and coaching are used together with teaching strategies that encourage students to articulate and reflect on their own thinking processes.

MOTIVATION AND PHILOSOPHY OF THE COURSE

The original starting point of the course was the observation that despite extensive traditional teaching, students have acquired a considerable amount of knowledge (in mathematics) that they are often not able to use. Why is this the case? There could of course be several reasons for this, including the possibility that the students simply haven't understood the contents. This is often the outcome of approaches to teaching that do not actively engage students, do not demonstrate the relevance of the material, and do not address student misconceptions (Ramsden, 1992).

While acknowledging these well-known problems, we want to highlight another problem that we believe is fundamental. It is that students have generally been taught to *solve well-defined problems by using given methods*, i.e. by starting with knowledge that somehow just exists, and applying this to specific problems that fit perfectly with the theory presented; Wedelin et al. (2014) discuss how this can be observed in the present course. This has many consequences. One is that students *know little about what real problems actually look like*. They also know little about *the early stages* of solving such problems, which typically include modelling and related activities of observation, judgement and simplification, which are difficult to formalize. Instead, while teachers often tend to avoid the fuzziness of such problems and activities, problems given to students typically begin with an already given model (Zawojewski et al., 2008).

Another consequence is that students do *not develop a balanced view on the relationship between knowledge and their own ability to think*. When students encounter a problem they do not know how to solve, they give up too soon, assuming that more (existing) knowledge needs to be learned in order to solve this apparently new kind of problem. As this repeats itself, due to the diversity of problems, students' ability to solve problems and their current knowledge always seems to be insufficient, leading to a constant lack of self-confidence.

The result is that students become followers, asking for solutions rather than thinking about problems and how to solve them. And in this sense they have in a way lost their own ability to think. In terms of Schoenfeld's theory, this leads to a lack of control, and creates beliefs and attitudes that are unhelpful for constructive problem solving.

In the course, we therefore wish to help the students to regain the power of their own thinking in the following ways:

- Addressing mathematical modelling (in some areas of engineering). We provide a basic familiarity with an important and non-trivial class of realistic and ill-structured real-world problems, where we also consider the often neglected early stages of solving such problems. The modelling connects applications with the mathematics that the students already know; this is an example of a significant knowledge base that students thereby learn to effectively access and make use of.

- Making the students aware of the power of *learning by exploration* (as opposed to learning by reading a book or asking someone). This is an essential component in any real problem solving. It requires that you take control of and make the most of your own thinking, and that you believe that you with considerable effort may be able to find something interesting. Understanding an unknown problem or situation requires that it is investigated in an exploratory learning process which involves asking questions, looking at the problem from different points of view, deciding what is relevant, and so on. It further involves exploring different ways forward towards any goal. This leads to a deeper understanding, necessary to be able

to effectively use what you know in new situations, where existing knowledge is typically useful as a part in the whole. Only little new knowledge is required for this (depending on the problem domain).

• Giving students the insight about what is required to develop a problem solving ability also in other domains. This includes a similar understanding in terms of familiarity with many cases in a domain, relevant background knowledge, specific heuristics etc. Taking control of your thinking and associated constructive problem solving attitudes will be important in any domain.

DRAWING ON A LARGE SET OF STRUCTURALLY VARIED CASES

In order to learn and teach mathematical modelling and problem solving for real-world situations, a sufficiently large set of problems is needed, exhibiting a representative variation around the critical aspects that need to be covered (Bowden & Marton, 1998). However, in science and engineering, real-world problems often correspond to projects of considerable size. This means that, even for relatively small projects and entire educational programs, it is difficult to fit in an appropriate number of problems for students to experience the essential variation in critical aspects. Moreover, the problems will be time-wise apart, so it will be difficult to compare and discuss problems side by side. On the other hand, too small and possibly artificial problems may not serve well as models for real-world problems. It is also important that each problem is sufficiently large to be remembered as a case that can be referred to in the course and beyond. The students then build a *case library* of problems that they have solved themselves, to draw on during the course and in their future problem solving activities in the engineering workplace; see the previous discussion on case-based reasoning (Kolodner, 1993).

One of the key ideas when designing the course has therefore been to create *simplified* but still *reasonably realistic* problems (Wedelin & Adawi, 2015), so that a sufficient number of problems fit in a single course. This is somewhat facilitated by our focus on the use of mathematics, since mathematics is often embedded in other time-consuming activities of a project. We can therefore often demonstrate how mathematics can be used in real-world situations without actually doing a whole project.

When designing the problems, we have considered three critical aspects of being able to deal with real-world problems. The variation around these three critical aspects can be seen as spanning three *dimensions of learning*, see Figure 1. These dimensions, for which we also discuss our criteria in selecting and simplifying problems, are:

• *Familiarity with real-world problems* – a realistic problem and its solution (including any necessary derivation), acts as a representative case and contributes to a familiarity with real-world problems in the domain of interest. By realistic we here mean a problem that is meaningful to solve because its solution is of genuine interest, practically or theoretically. With this definition we include not only applied problems of immediate practical interest, but also problems related to steps in the formation and explanation of relevant theory or methods – such problems are useful to better understand theories, and also serve as examples of useful subproblems even if the end goal is to solve an applied problem. But we exclude artificial problems for which the sole meaning is to test if a given concept is understood or to practice some particular method, which is abundant especially in mathematics but also in other areas of science and

engineering. The challenge is then to keep the essence of the real-world problem: the simplified problem should still be a reasonably truthful model of some real problem that is actually out there, and any solution, its derivation, and its interpretation in the context should be similar.

- *Supporting knowledge* – the concepts and methods required to solve the problem (known in advance or created as a part of the solution process). Whether real-world problems require a lot of new knowledge to be learned clearly depends on the situation. We have here decided to avoid problems that require too much new theory to be learned, in order not to obscure our goal of developing the students' own thinking and problem solving skills, and not to make them believe that the purpose of the course – and the way to become a better problem solver – is to learn more new theory (as in most other courses).

- *Processes and skills* – the particular way in which the solution (and its derivation) was found, and the modelling and problem solving techniques involved. This typically involves a search process where different considerations were made and different approaches were attempted until the suggested solution was found. In order to ensure that any simplification does not go so far that essential real-world characteristics are lost in this important respect, we have calibrated the problems so that they should not be easy to understand, and they should be challenging to solve. This, in other words, corresponds to the idea that the problems should be at least somewhat ill-structured, to invoke genuine modelling, problem solving, and the need to communicate.

It then also makes sense for the students and the teacher to discuss and reflect on the problems with respect to these three dimensions. This includes the specific context of the problems for each dimension, as well as what the problem as a representative example can convey about each dimension in general. We refer to this entire discussion and reflection as a *perspective*, and consider it to be an integrated part of the problem although it may not be a part of the problem statement. In the perspective we include relevant insights, attitudes, and expectations.
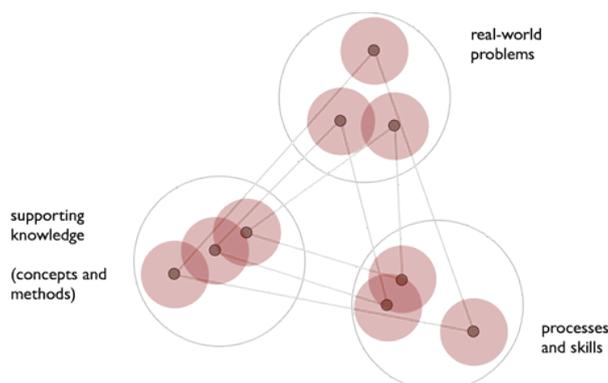

*Figure 1. Illustration of how several problems and their perspective span
the three dimensions of learning, where each triangle represents one problem*

Turning to how to design the set of problems, the main issue is that the problems have to be carefully combined so that they together comprise a sufficient and representative variation in all three dimensions of our model. In other words, that the set will be useful to create familiarity with the real-world problem domain in question, to highlight the different kinds of knowledge required, and to develop basic problem solving skills for problems in this area. Exactly what it means to cover a dimension is difficult to formalize, in that it is a combination of many detailed conscious and intuitive considerations of what is useful to include. The global view also allows us to be pragmatic in the application of our general criteria for selecting problems, and if some problem or sub problem serves a sensible purpose in a bigger whole we do not hesitate to include it.

In total we have formulated about 30 problems broadly within the area of mathematical modelling, which we have grouped thematically after model type in six weekly modules. We here present three examples of problems from the course:

- Students are asked to find a good curve to fit a number of given points (time and distance). This is a problem given just in the beginning of the course. No prior instruction about curve fitting or the least squares method is given, and it is up to the students to come up with some way to approach the problem, determine what is a good fit etc. There is a very good answer to this problem, which is Kepler's third law, but other solutions are equally accepted. The perspective of this problem includes the nature of different mathematical functions, curve fitting problems in general and where such problems arise (including a little history of science), different methods for curve fitting such as the least squares method, a reflection on what is "good enough", and how it is natural to work with this problem in an iterative way, finding successively better models – essentially the so called *modelling cycle* (Borromeo Ferri, 2006), which the students through this problem discover for themselves.

- Students are given a description of a simple road network between two cities along a curved river, with certain given mathematical assumptions about how travel time on some roads changes with traffic intensity. We are concerned with the travel time between two cities, and the question is to what extent travel time could be reduced by building a bridge. An analysis includes thinking in terms of traffic flow and equilibrium, and requires assumptions about driver behaviour. It turns out that the bridge actually *increases* travel time; this is known as Braess' paradox. The perspective of this problem includes how simple mathematical examples and models can highlight and explain unexpected phenomena which exist in reality but are difficult to see, Nash equilibria and its wider consequences, how flow and equilibrium are useful simplifying modelling concepts, how relevant assumptions can be made, and how you may fail if you are unable to find a precise mathematical formulation (in this case the criterion for the equilibrium). The problem involves both significant qualitative and quantitative thinking, but the actual calculation is almost trivial – so it is also an illustration of that more advanced math is not the way to solve this problem.

- The question is how you can determine the dosage, and time between doses of a drug. This involves understanding the problem both quantitatively and qualitatively, deciding what is relevant, making considerable simplifying assumptions, suggesting what you need to know etc. A key step is to model how the concentration of the drug in the blood varies with time. The problem can be solved in several ways, from very complicated and exact to very simple

but with unrealistic dangerous consequences. The perspective here includes the considerable modelling simplifications that are possible by thinking about the context and using common sense knowledge, without knowing very much about the specific problem, and the importance of a qualitative understanding of the quantitative behaviour both of reality and any models. In the problem solving, it is about how you can easily get stuck in a complicated mess, about clearly structuring your problem solving in suitable steps, starting with something simple, being careful and drawing the full logical consequences of your own assumptions, and see the many detailed ways - easy and complicated - in which the problem can be solved.

For a more detailed formulation of these problems, and more problems, see Wedelin (2015).

TEACHING PROBLEM SOLVING IN A COGNITIVE APPRENTICESHIP ENVIRONMENT

The learning environment that we have designed for the course, of which the problems constitute a key element, can be characterized as a *cognitive apprenticeship* environment (Collins, Brown & Newman, 1989) where students practice solving reasonably realistic problems where no specific method to solve them has been explained, and where the teachers actively in lectures and in supervision attempt to share their experiences and insights in modelling and problem solving. This also encourages attitudes and work practices similar to those of people who have experience in handling real-world problems.

Students work in groups of two, to ensure that everyone is actively engaged in the problem solving, and to enable communication as an important aspect of the problem solving process.

Every weekly module – six in total – begins with an introductory lecture introducing the theme for the week, however without explaining how to solve the problems. After each week there is a follow-up lecture presenting the solutions of the teacher, common difficulties and alternative approaches, and a perspective on the problems solved and beyond. The course ends with a reflective report.

The course is well-structured in the sense that students start working from the first day and roughly know what they are expected to do every week, and with many problems and weekly modules there is a sense of continuous progress. The course offers a controlled environment also for the teachers, since the relatively small problems can be supervised in a reasonably predictable way, and useful supervision guidelines can be written to help new course assistants.

Extensive opportunities for supervision are provided, where the limited size of the problems enables practice and feedback on the whole problem solving process many times during the course. The challenging nature of the problems makes communication within the group and with the teacher necessary. Teachers give Socratic supervision – asking questions rather than giving answers – and in context give general advice about how to solve problems. The problems in combination with this kind of supervision turn out to be efficient in identifying individual student difficulties, and make it relatively easy to give appropriate feedback (since these smaller problems in this controlled and predictable setting are relatively easy to learn to supervise).

There is a strong emphasis on the process of solving problems, rather than on solving problems "correctly", or to a given level (it is not always clear what the "best" solution is for modelling problems). Failure is seen as normal, and a learning opportunity. Instead, everyone is encouraged and expected to do their best. As a consequence, assessment is pushed to the background, and students are confident that if they work hard they will pass. We promote the adoption of

constructive problem solving attitudes, including patience, self-awareness and that the students should trust their own thinking, so that students grow into the role of being a problem solver.

Throughout the course we encourage reflection, and in particular what students can learn by comparing what they did themselves with the solutions suggested in the follow-up lecture. We try to exploit the variation in the problems by discussing relevant differences and similarities across problems. We also use the perspective to add to the students' own experiences by pointing to relevant higher order issues in modelling and problem solving, including highlighting important classes of problems, and general ways to think and talk about modelling and problem solving. This means that the role of the teacher is not only to have subject knowledge or know how to solve problems, but also to talk about the problem solving process, and raise awareness about key issues. There is no magic in this, but more of an ambition of the teacher to share his experience to reach beyond the individual problems that the students have worked with.

For a more detailed description of the design of the learning environment, see Wedelin and Adawi (2014).

DISCUSSION

The course shares with PBL the idea of inquiry-based learning, and the ambition that students through this kind of instruction should learn to solve unknown real-world problems. However, the course is also quite different in its use of *many smaller problems rather than a few big ones*. This, by itself, creates a natural step in a progression. It also enables a more continuous variation in all aspects of the problems, and therefore an increased opportunity to see patterns, especially in the problem solving process. We can then be more active in helping the students to see such patterns, and explicitly teach problem solving based on this variation. Scaffolding is considered very important in PBL, but is mostly focused on solving the problem given to the students, rather than teaching problem solving. At least this does not come automatically from how PBL is commonly described.

We note that if PBL is applied to domains where the practice naturally consists of many cases, no major effort may be required to make problems or cases smaller. However, PBL is not explicitly based on learning to handle diversity by variation, which is important for problem solving in any domain. A lack of diversity in PBL has been observed in the phenomenon that subject coverage becomes more difficult to achieve compared to traditional teaching. However, given the model in Figure 1, it appears that the same difficulty could also apply to the other dimensions of learning. So it is important to remember that a pattern can only appear when you are provided with several examples. And it is only possible to make the necessary judgements if you are familiar with the problem domain, in terms of the diversity in problems previously encountered and solved.

Then, while students have to solve problems, without knowing in advance how to solve them, *we otherwise try to reduce the complexity* around this challenge. And given our students' lack of experience in basic problem solving this is already a major step compared to working with given methods. The students do not need to take major control of planning of activities or their own learning, before they are used to take control of their thinking, and have a balanced view between the trade-off between learning by thinking, and learning by studying. However, they do have to keep track of how they spend their time, which is important in all problem solving. The smaller problems facilitate a well-structured course, make small two-person groups natural, and simplify effective feedback and supervision. Finally, learning new traditional knowledge in terms

of conventional theory is not a significant part of the course, and would be counterproductive in our desire to change students' beliefs about what you need to solve problems. It is also quite unnecessary, since our students already have a large body of knowledge in mathematics, and we find it natural to make sure that the ability to use what you know should have a priority over acquiring new knowledge.

Turning again to PBL, we find that PBL addresses a more ambitious goal that should come later. Of course, mature students and professionals should be able to handle the full complexity of large real-world problems, but if a fundamental and identifiable gap in students' abilities can be observed, it makes sense to target this specifically. When students know how to take control of their thinking, and have a balanced view between learning by thinking and learning by studying, it makes sense to continue to more ambitious tasks.

CONCLUSIONS AND FINAL REMARKS

In this paper we have described the design of a course in mathematical modelling and problem solving for engineering students. The course aims to prepare students for handling unknown future problems and situations to a greater extent than traditional courses. Both the modelling aspect and the problem solving aspect of the course contributes to this. This also requires some significant unlearning, caused by prior ways of teaching, where we note that a key missing component in students' ability to solve problems is *learning by exploration*. The focus on modelling problems further addresses the early stages of real-world problem solving, which is likewise neglected in traditional engineering education. Another effect of the course is that modelling and problem solving become real topics for students, the level of abstraction is raised, and a more process-oriented view is developed.

A particular feature of the course is the relatively large set of problems, which creates an opportunity for the students to experience a range of problems that in the case of full size problems would take a very long time to accumulate. This is therefore a complementary experience to what PBL typically offers, at least in some areas. Furthermore, the problems give the teacher an opportunity to convey her experiences related to the variation across problems, which cannot be naturally conveyed if only a few large problems are considered.

The learning objectives have been carefully chosen, and the learning environment has been designed uncompromisingly based on these. Our experience with the course shows that these objectives are indeed very important for students who have been raised in a traditional learning environment. We also see that the students are able to adapt and relearn very quickly in the right environment (Wedelin et al., 2014). Problem solving abilities, including student beliefs and attitudes, are significantly developed within the scope of a single course, and students adopt a problem solving mindset. The response from the students is exceptionally positive. Many students say that they have developed a new way to think. We receive comments like "we have been able to solve problems that we thought we would never be able to solve", and "Can you really learn problem solving, learn to see patterns? In my own experience with this course, yes. It is not a talent, it is a way to think that you can learn".

Clearly, the need for this kind of teaching depends on what the rest of the curriculum looks like. For students with a background in traditional engineering courses it is very important. In a program dominated by a PBL method of instruction, the course provides a progression, with a more constrained mode of teaching, and by requiring little in terms of new theory. It also provides concepts and a language for talking about modelling and problem solving. It is also

complementary in that experience from a sufficient number of problems or cases is fundamental to developing a problem solving ability.

The course itself can be of interest for a wide range of educational programs. Mathematics is one of the few significant bodies of knowledge that students in many different programs have. There is therefore a certain generality to this teaching, and with some adaptations in the selection of problems, a similar course can be given to students in many different programs.

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